Morphological Metrics in a Bounded Space

By Andrew C. Smith, 2013

0. Introduction

In *Topology (phases of this difference)*, my goal was to create a compositional system that generated pieces deterministically, based on the latitude and longitude of a given performance venue. However, there are a number of qualities that all versions of the piece share. They each consist of 29 nine-note melodies played in hocket by two violins. The intervals between pitches are tuned in just intonation, ranging from 8/7 (a wide major second) to 8/1 (three octaves), drawn from a set of intervals tuneable by ear outlined in Sabat (2004). Only the central melody (#15) stays the same from performance to performance, while all others shift depending on the latitude and longitude of the venue. Using the morphological metrics described in Polansky (1996), the 28 changing melodies are found by transforming the central melody until it is a certain "distance" away from its starting point. The first 14 melodies gradually move closer to the central melody, while the last 14 move further away. Thus, the piece is coherent from performance to performance—as in, the same basic actions take place, using the same instruments, and the piece is roughly the same length with the same figure in the middle—and yet the content of the composition changes with each new location.

This paper outlines a method of measuring distance and angle with morphological metrics, using my piece *Topology (phases of this difference)* as an example. Fundamental to this project is the creation of a piece that contains a range of possible performances, rather than a fixed work unto itself. This way, the final note-to-note decisions are not left up to the composer (or even to the performer)[1] but are meant to reveal an aspect of the generative machine behind the work. With each new performance location, a new expression of this machine is articulated.

The piece uses a polar coordinate system, where the pole (or "center" point) is a melody written by hand, provided as input to the composition. The 14 melodies in either half of the piece are defined by their overall distance and angle from the pole. The magnitude is found by taking the square root of the sum of sqaures of metrics measuring harmonic distance and melodic distance, while the angle is the ratio of harmonic distance to melodic distance. The angle of approach is determined by the longitude of the performance location, while the initial distance from the center is determined by the latitude (i.e., the greater the "slice" of Earth in the latitude—the closer the performance is to the equator—the further from the center point the first melody will be). The angle away is rotated π radians (180°) from the angle of approach. Essentially, this means that a string of variations should be perceived as travelling through the central pole, and retreating away on the same trajectory.

I. Similarity and difference of melodies from a single melody

A melody might be dictated by providing precise notes—C4, D4, A3, G4—but it could just as easily be described in terms of its intervals: begin on C4, move up a major second, down a perfect fourth, and up a minor seventh. Each of these intervals has a particular magnitude—a fourth has a lesser magnitude than a fifth, which has a lesser magnitude than a minor seventh—and a contour of up, down, or the same. This is a way of describing a melody as a series of transformations (or intervals), rather than as a series of fixed points. Straus (2005) discusses a pitch interval in terms of the number of semitones traversed, which is essentially a geometric relationship since identical ratios are identical intervals even if the arithmetic differences between their frequencies increase. The formula used for this common pitch distance (PD) metric is log(p1 / p2), where the two pitches p1 and p2 are expressed as frequencies.[2]

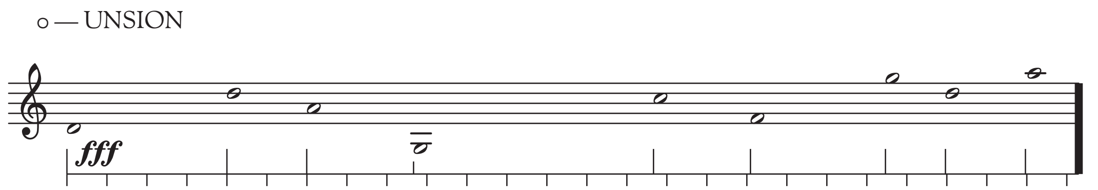
A second way of thinking about the magnitude of intervals, however, is in harmonic distance (HD). Many HD functions seem to share the intuition that intervals based on lower prime numbers (e.g., a perfect fifth tuned as 3/2, a fourth tuned as 4/3), are closer harmonically than intervals based on higher primes (e.g., 17/16, or about a semitone). An HD function for 12-tone equal temperament might be based on the number of movements around the circle of fifths required to travel from one pitch to another.[3] In just intonation, there

1 This is different from some indeterminate works, which are indeterminate by the basis of allowing the performer to make decisions either in the preparation or performance of the work.

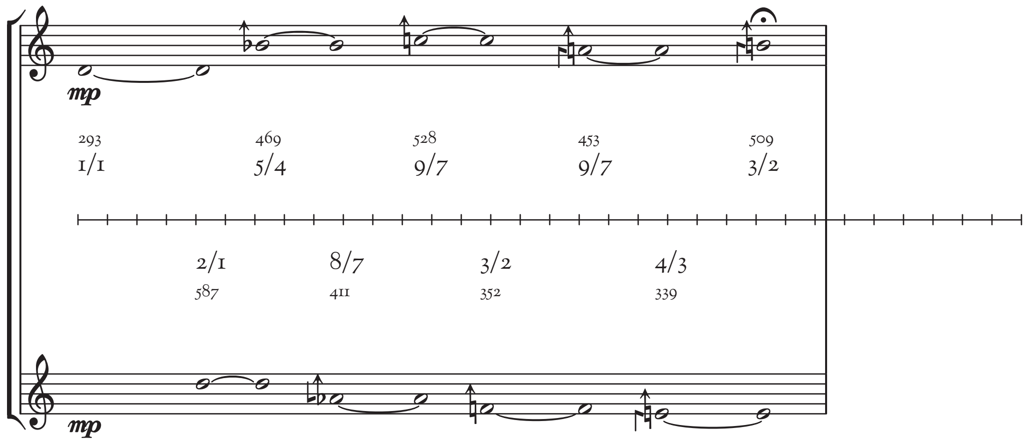
2 Most commonly, a base-2 log is used, which leads to an "octave" that equals a distance of 1. Multiplying the result by 1200 gives the traditional definition of cents, where 100 cents is equivalent to a 12-tone equal tempered semitone.

3 In this case, the harmonic distance from C to G would be 1, C to D would be 2, C to F-sharp would be 6, etc. This is

A:



B:



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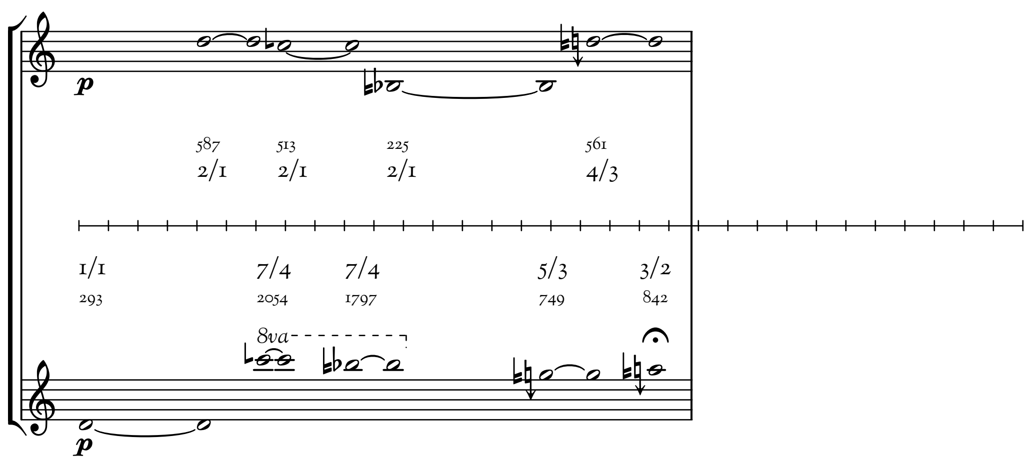


Fig. 1: Plotting points as distances from A

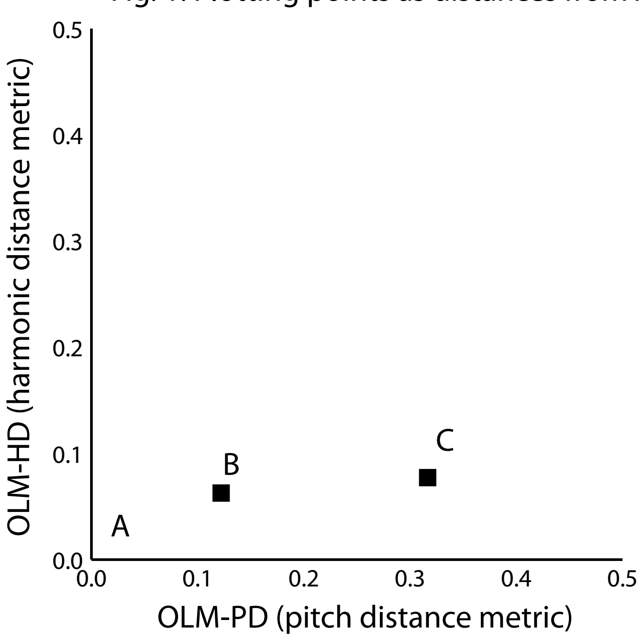
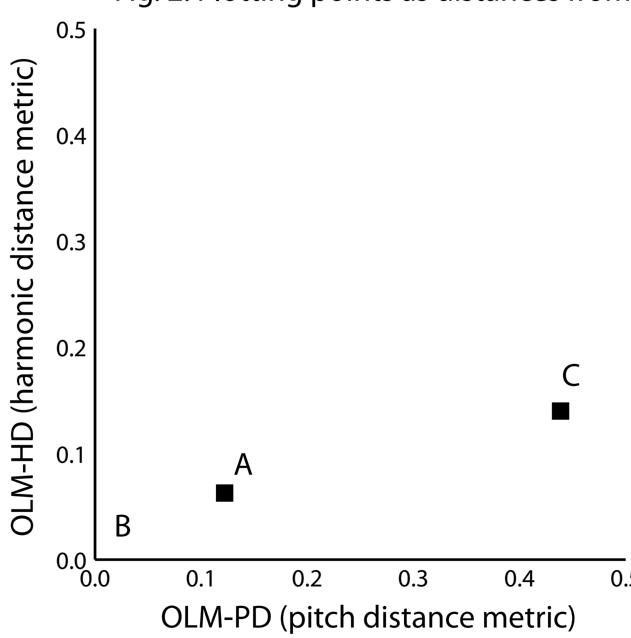


Fig. 2: Plotting points as distances from B



are a number of established HD functions (Chalmers 1993; Barlow 1987; Tenney 1987) but for various reason this piece uses the Tenney HD function with a base-2 logarithm.[4]

After abstracting a melody into a sequence of intervals, it is possible to consider the degree of similarity two melodies—the distance described above. The morphological metrics cataloged in Polansky (1996) have proven to be a useful tool for quantifying the difference between melodies, because of their focus on key musical parameters such as interval magnitude[5] (the distance from one pitch to the next) and contour (which direction). The simplicity of these metrics lend themselves to measuring some of the most basic things we notice about music in a way that is common to rhythm, melody, and larger-scale forms: a melody can go up, a rhythm can accelerate, and a piece can be built from gradually shorter and shorter phrases.

The metric used as an example throughout this paper is the ordered linear magnitude (OLM) metric, which measures the average difference between the magnitudes of corresponding intervals in two interval sequences of the same length. Essentially, this judges whether the leaps in one melody are similar to the leaps in the same position in another melody.[6] Morphological metrics use a delta function to define the notion of "difference," determining what aspect of the melody the OLM measures. It is possible to use the HD and PD functions as two separate delta functions (OLM-HD and OLM-PD), plotting the two metrics on two axes, as a way to visualize the OLM-HD and OLM-PD of multiple melodies from a single fixed point [Fig. 1]. This plot allows for the visualization of melodies that are similar in one way but not in another. (Changing every one-octave leap to a two-octave leap is almost inconsequential to the HD function, but it is a great difference for the PD function.) These two distances are converted into the polar coordinate vector described above: the magnitude (using the squared root of the sum of squares), and the angle (using the ratio of the OLM-HD to OLM-PD).

However, two points could look like they are a certain distance away on this initial graph, but measuring the OLM-HD and OLM-PD between them might show that these two points are [Fig. 2]. This is because the graph is only a graph of distances in two dimensions, essentially similar to a top-view two-dimensional geometric projection of a rectangular prism, where the lack of depth leads to the illusion that two corners occupy the same space when they may be any distance apart. Graphing these melodies against a fixed point is a convenient way to measure the similarity of a range of melodies relative to one fixed melody, but it does not provide a measurement for the other melodies to one another.

II. Directionality of the vector

Thus far, the notion of "distance" from the central melody, and the definition of angle solely as the ratio of OLM-HD to OLM-PD, only allows for vectors that are positive—essentially, those with angles between 0 and π/2 radians, which are in only one quadrant of the polar graph. This would not allow for the desired feeling of moving "through" the pole, as described above, which was one of my primary goals in constructing the piece. Therefore, there must be a sense of directionality added to the result of the OLM metrics; it needs to be different *in a specific direction*. Essentially, each of the two OLM metrics is its own dimension, and the formula for converting these values into a magnitude on the polar graph is the Pythagorean formula. If a third point—the "edge"—is defined in a given dimension, it becomes possible to think of the four quadrants as a hybrid of the polar and Cartesian coordinate systems, and to determine the angle of rotation based on the variation's relationship to the edge. If the variation's distance from the edge is greater than the center point's distance from the edge, then it is in the half of the graph further from that edge. Apply this again using the other metric, and the result is narrowed down to a single quadrant. The angle (determined by the ratio of OLM-HD to OLM-PD) is then rotated π/2, π, or π + π/2 radians until it is in its correct quadrant.

The clear choice for the edge of both dimensions was the sequence [1/1, 1/1, 1/1, 1/1, …], because of the audible structure that it gave the piece; it could be called the "least com-

similar to the concept of related and distant keys in tonal music analysis; a modulation to a key one semitone away would be considered more distant than one to the dominant key.

4 A discussion of the relative merics of all the metrics mentioned is beyond the scope of this paper, but the basic reason is that Tenney's pitch distance metric, in its log2(x * y) form, is similar in effect to the pitch distance metric used. Each successive harmonic (2/1, 3/1, 4/1, 5/1, etc.) will have the same distance in both metrics, as long as both metrics use a base-2 logarithm.

5 Polansky's use of magnitude regarding intervals is distinct from the "vector magnitude" judging the distance of one variation from the central melody. I will use "interval magnitude" to describe Polansky's notion of the leap from one interval to another where possible.

6 The OLM does not judge contour, which can be a deciding factor in the perception of melodic similarity. A melody's intervals may be the same, but the contour may be completely different. Think of a circle of fifths versus a repeated C-G-C-G-C…; both contain nothing more than a fifth, but one keeps going up while the other alternates. For this reason, each variation in *Topology (phases of this difference)* used the contour with the contour that was most similar to the center melody.

Fig. 3: Plotting +/- of points

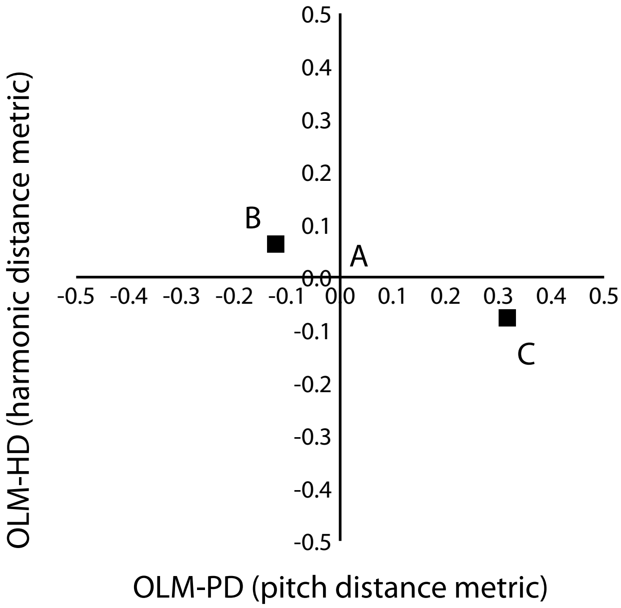
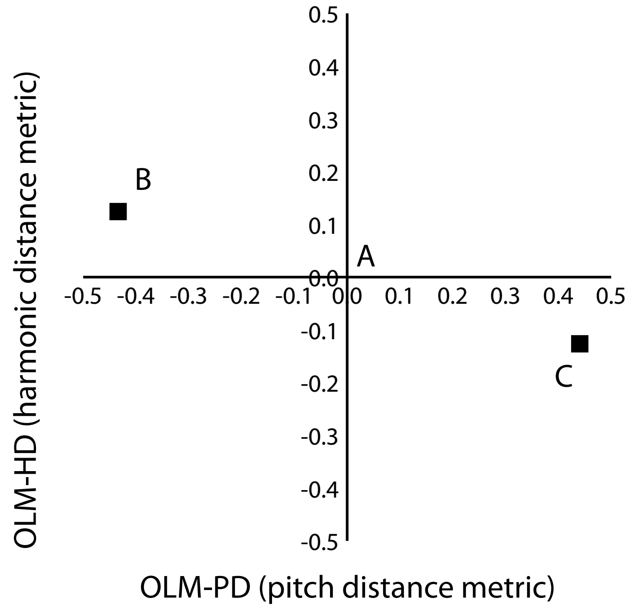


Fig. 4: Scaling each quadrent to +/- 1.0



plex" melody in either dimension. This means that if the first fourteen melodies are less harmonically complex than the center melody—which is to say, closer to the [1/1, 1/1, 1/1…] sequence than the center melody is—then the last fourteen will be more harmonically complex. The transformed melodies are plotted in [Fig. 3].

III. Scaling the dimensions

This method faces one major issue: the notion of scaling each dimension, so that both dimensions are equally important. Of all the scaling methods outlined in Polansky (1996), the most relevant is called "absolute" scaling, where the interval magnitude of each interval is scaled so that the greatest interval magnitude in either melody is 1.0, and all other interval magnitudes are scaled accordingly.[7] The intuition behind absolute scaling is that any interval occurring in one melody could have occurred in the other melody as well. Likewise, it lends itself to extension across more than two melodies. [Fig. 4] shows the three melodies scaled linearly such that the greatest delta across all three melodies is 1.0. This presents one major problem, where if a newly introduced interval contains a greater delta function, the entire collection of distances must be recalculated. This issue is solved by using a fixed set of intervals from which all the melodies are derived, finding the greatest possible interval magnitude that could occur across any of these melodies, and scaling to this value.[8]

This was very important for the piece for one primary reason: each performance in a distinct geographical location must use a distinct version of the score. I wanted each piece to be judged relative to all possible pieces, so that a performance near the North Pole would have a smaller range of HD and PD than a performance near the equator.[9] Scaling both dimensions to the absolute boundaries of all possible melodies helps to maintain consistency across performances and versions of the piece. At its core, this notion of scaling provides a method of judging objects relative to a collection, rather than relative only to one another.

IV. Ending

The method of plotting melodies as points, used in the

7 Since the above-mentioned pitch distance and harmonic distance functions already use the logarithmic scale, I use linear scaling, dividing each delta by the maximum.

8 In addition, the center melody is scaled to be exactly 0.5 from the edge in either dimension. Therefore, all points closer to the edge than to the center melody are divided by the distance of the center melody from the edge, and all points further from the edge are divided by 1 - this distance, and both sides are multiplied by 0.5.

9 In fact, performances at the North Pole and the equator are practically impossible because of limitations on the possible melodies in the space. The initial distance from the center melody grows as the distance from the equator grows, and the angle of approach corresponds to standard longitude from the Greenwich Observatory. The N/S axis is PD, while the E/W axis is HD.

composition of this piece, could provide useful insights for analysis as well as composition. For example, tone rows of individual 12-tone serial pieces might be examined in terms of all rows ever used by a given composer, allowing for the quantification of the similarity of individual tone rows relative to a composer's entire output. A similar project could be focused on fugue themes, a collection of cantus firmus melodies from a given period, the set of rhythmic modes, or any other collection of sequenced musical objects. Further projects might take into account a greater number of simultaneous metrics in higher dimensionalities, as well as additional musical parameters and multimetric functions. Essentially, this is a way of projecting a set of complex multidimensional relationships—the distances of all melodies from all other melodies—onto a simple two-dimensional graph.

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References

Sabat, M. (2004). Analysis of Tuneable Intervals on Violin and Cello. http://www.marcsabat.com

Barlow, C. (1987). Two essays on theory. Computer Music Journal, 11(1), 44-60.

Chalmers, J. (1993). Divisions of the Tetrachord. Lebanon: Frog Peak Music.

Tenney, J. (1987). About "Changes": 64 studies for 6 harps. Perspectives of New Music, 25(1, 2), 64-87.

Polansky, L. (1996). Morphological Metrics. Journal of New Music Research, Vol. 25, 289-368.

Polansky, L. & Bassein, R. (1992). Possible and Impossible Melody: Some formal aspects of contour. Journal of Music Theory, 36(2), 284.

Straus, J. (2005). Introduction to Post-Tonal Theory, 3rd Edition. Pearson Education: Upper Saddle River, N.J.